

Topology of full QCD *

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We study topological properties of SU(3) gauge theory using improved cooling. In the absence of fermions, we measure a topological susceptibility of $(182(8) \text{ MeV})^4$ and an instanton size $\sim 0.6 \text{ fm}$. In the presence of light staggered fermions and across the chiral transition, the susceptibility drops in a manner consistent with the quenched case, and the instanton size changes little. No significant formation of bound instanton-antiinstanton pairs is observed, in particular not along the Euclidean time direction for $T > T_c$.

Here, we extend to SU(3) our earlier study of topological properties of the SU(2) vacuum using improved cooling [1]. The topology of lattice gauge fields is obscured by lattice artifacts (dislocations). We remove them by cooling. To avoid losing at the same time physical, large-scale topological structures, we cool with a highly improved action. This action provides a small energy barrier against the decay of instantons of size $\rho \gtrsim 2.3a$, thus guaranteeing stability of the measured topological charge under *arbitrary* amounts of cooling. Furthermore, it gives a scale-invariant instanton action, up to terms $\mathcal{O}(a/\rho)^6$, ensuring good stability of the instanton size under cooling. Using 5 planar loops (1x1, 2x2, 3x3, 1x2, 1x3), we construct a 1-parameter family of such classically improved actions, then tune this parameter for maximum scale-invariance. We use for SU(3) the same action as for SU(2), with the same tuning coefficient. Because an SU(3) instanton is constructed by embedding an SU(2) instanton, and because cooling preserves this embedding, there is no need to retune the SU(3) action.

1. Pure gauge

We have applied improved cooling to SU(3) gauge theory at $\beta = 5.85$ (12^4) and 6.0 (16^4),

on 120 configurations each, separated by 50 MC sweeps (in a 4:1 mixture of OR and PHB). The MC time history of the topological charge Q at $\beta = 6$, shown in Fig.1, reveals autocorrelations already known to plague full QCD simulations. Since topological sectors are disconnected from each other in continuum field theory, energy barriers of height diverging as $a \rightarrow 0$ must be present in the lattice theory (full or quenched QCD). The autocorrelation of Q is a clear indication that we are measuring a relevant physical quantity: any contribution to Q of lattice artifacts (eg. dislocations) would appear as additional noise and quickly wash out the autocorrelation.

The resulting topological susceptibility χ is $(185(7) \text{ MeV})^4$ [$\beta = 5.85, a = 0.134 \text{ fm}$] or $(182(8) \text{ MeV})^4$ [$\beta = 6, a = 0.1 \text{ fm}$], in good agreement with Ref.[2]. The size distribution we measure (see Fig.2) scales well with β . A slight shift to larger sizes is observed under cooling, and fits of the form $\rho^6 \exp(-(\rho/w)^c)$ are somewhat cooling-dependent (see figure). However the peak of the distribution stays around 0.6 fm . Just like for SU(2) where the distribution peaked around 0.43 fm , we observe a significantly larger size than expected from instanton liquid models (cf. also talk by D. Smith). Together with the observed instanton density, it makes a dilute description of the instanton fluid rather questionable.

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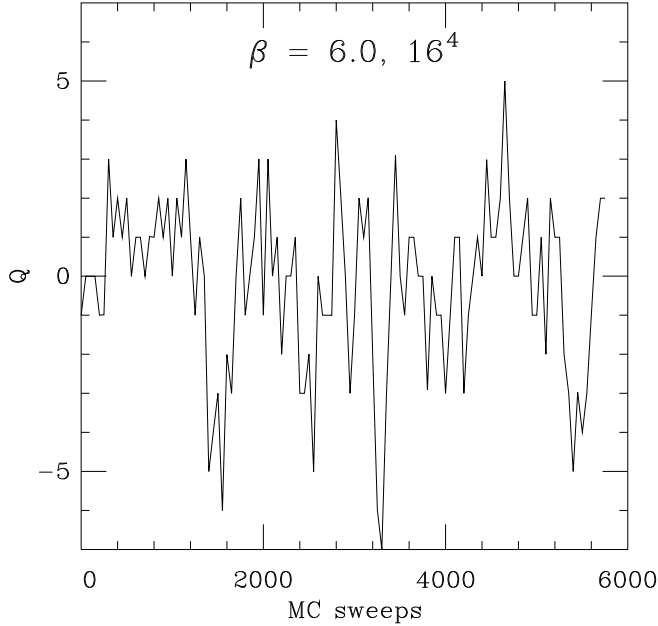


Figure 1. Evolution of the topological charge in quenched Monte Carlo.

2. Full QCD

We have analyzed configurations generously provided by the MILC collaboration [3]: $24^3 \times 12$, 2 flavors of staggered fermions, $ma = 0.008$. The dynamical quarks are thus rather light, with $m_\pi/m_\rho \sim 0.6$. Our sample consists of 31, 43, 20 configurations respectively at $\beta = 5.65, 5.725$ and 5.85 , corresponding to $T/T_c \approx 0.95, 1.05$ and 1.25 . Ergodicity problems are glaring at $\beta = 5.65$, since we measure $\langle Q \rangle = -1.68 \pm 0.33$ over the complete simulation run by MILC. At higher β , $\langle Q^2 \rangle$ is smaller, making the lack of topological tunneling less important. Ignoring these problems, one can infer the following values for the topological susceptibility, based on the measured values of $\langle Q^2 \rangle$: $(134(10) \text{ MeV})^4$, $(102(5) \text{ MeV})^4$ and 0 at the 3 temperatures (at $\beta = 5.85$ all 20 configurations have $Q = 0$). Thus the dynamical quarks suppress χ at the lowest temperature as expected; further suppression as a function of T/T_c appears consistent with the pure gauge case [2].

The extraction of instanton sizes should be modified from the $T = 0$ case: in the contin-

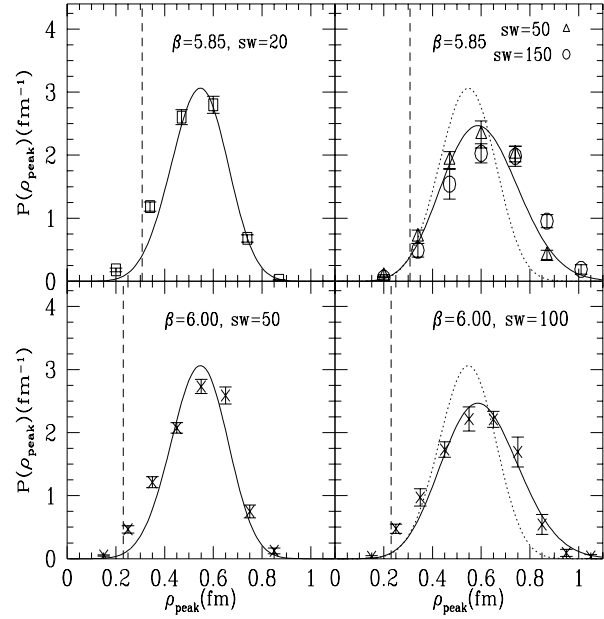


Figure 2. Size distribution for quenched SU(3).

uum at finite T , an instanton is distorted by periodic T -images into a “caloron”, and these distortions become important for our case where $\rho T \sim \mathcal{O}(1/2)$. Ignoring these effects for the time being, the typical instanton size observed remains $\sim 0.6 \text{ fm}$, with little dependence on temperature.

Instantons have been argued [4] to play a determining role in the chiral phase transition. The fermionic determinant induces an attractive force between objects of opposite topological charge, which thus tend to form dipoles; according to the instanton-liquid model at $T \sim T_c$ the attraction increases and free charges become rare, which explains the suppression of topological fluctuations; moreover these dipoles would align predominantly along Euclidean time. To check these predictions, we have measured the correlation $\langle q(\vec{0}, 0)q(\vec{x}, t) \rangle$ of the topological charge density as a function of the space and time separations \vec{x} and t . This observable can be measured at any level of cooling or with no cooling at all, and it bypasses any possible bias in the identification of instantons, which becomes delicate and ambiguous at short cooling. Some negative correlation is observable at short distance under short cooling;

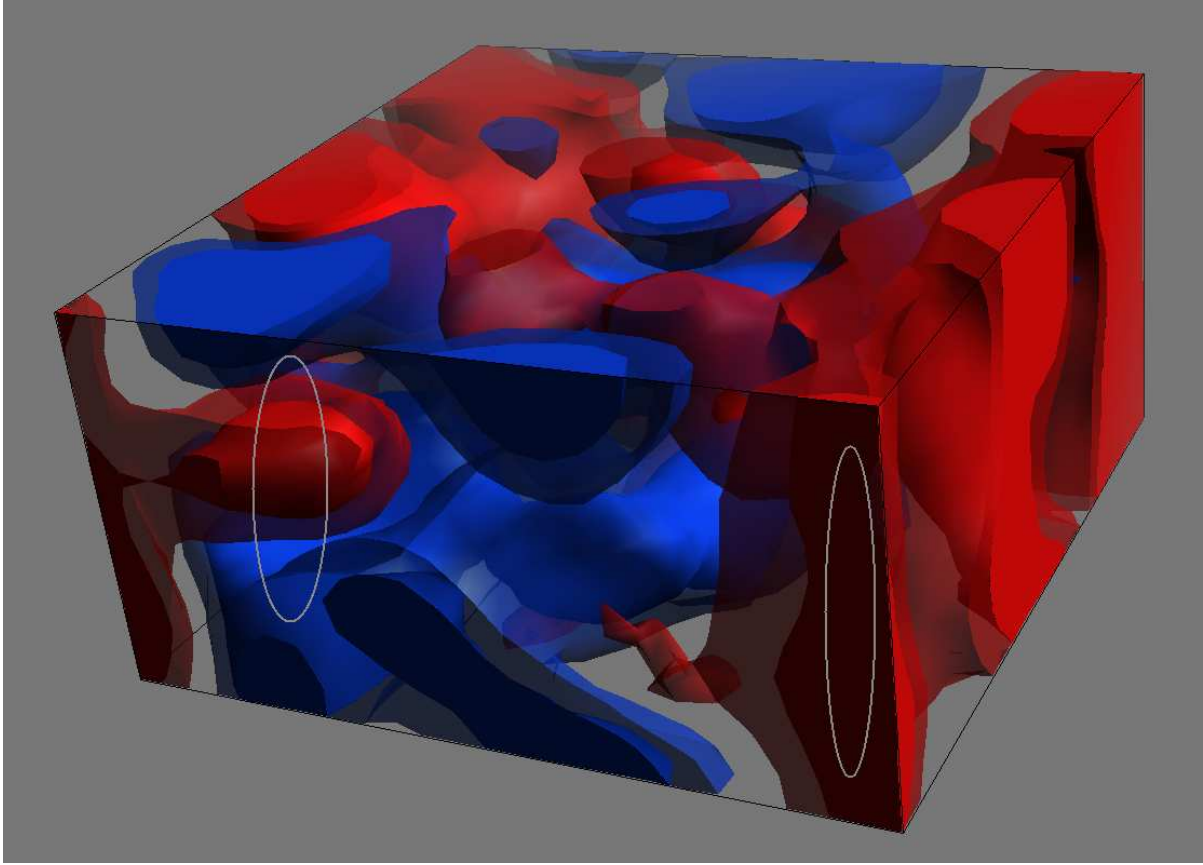


Figure 3. Isosurfaces of topological charge density ($\beta = 5.85$, i.e. $T/T_c \approx 1.25$) after 20 cooling sweeps. The left ellipse indicates the presence of an instanton-antiinstanton pair oriented along Euclidean time (the short direction), as predicted by the instanton liquid model. The right one shows a single static instanton. The solid and transparent surfaces correspond to a density of ± 84 and ± 34 ($8\pi^2$)/fm⁴ respectively.

the space-time anisotropy of these negative correlations is usually weak, $\mathcal{O}(10\%)$ or less. Reflection positivity enforces negative correlations starting at some distance in the absence of cooling, so it is hard to decide whether what we measure is really caused by genuine instanton-antiinstanton pairs. To gain some qualitative insight, we monitored isosurfaces of topological charge density $q(\vec{x}, t) = \text{constant}$. Fig.3 shows that two effects are in place: some alternating \pm charges at the same spatial location, in accordance with the instanton liquid scenario (left of figure); and for the most part, large single static charges, expected in other models (eg. [5]) (right of figure). One may argue that the quarks in the present simulation

are still too massive ($m_\pi/m_\rho \sim 0.6$) for the predictions of the instanton liquid scenario to apply.

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